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RESEARCH DEPARTMENT



REPORT

**DIGITAL LINE STORE STANDARDS CONVERSION:
preliminary interpolation study**

No. 1972/28

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DIGITAL LINE-STORE STANDARDS CONVERSION: PRELIMINARY INTERPOLATION STUDY

Summary

The experimental Digital Line-Store Standards Converter¹ built in Research Department served to demonstrate the good picture quality obtainable with digital techniques, even when using a comparatively simple linear interpolation aperture utilising information from only two lines of the input picture. The work described in this report was a study, mainly theoretical, into the requirements for a more sophisticated interpolation system involving the use of signal contributions from more than two lines. Computer optimisation techniques were used in determining the most suitable aperture shape using contributions from different numbers of successive field lines, and a block schematic diagram was derived for a proposed experimental interpolator to be used with the experimental digital line-store converter. A series of subjective tests is in hand to establish the basic interpolation parameters of a converter for service use. The results of these tests will be given in a further report.

1. Introduction

The use of digital techniques in standards conversion offers the potential advantages of high reliability, small size, reduced cost, and, provided that the arithmetical operations involved in signal processing are correctly carried out, optimum picture quality from the output signal, however complex the signal processing. This last advantage is of particular importance when considering the requirements for interpolation, since, as a rule, the more lines which are available simultaneously, the more accurate will be the estimate that can be made of the picture information that would have been produced had the signal been originally scanned using the output standard.

Information from a block of lines can be made available simultaneously by using the signals appearing at the inputs and outputs of a number of one-line delays in series; analogue signal delays of this duration produce small but perceptible picture defects. However, shift registers provide a means of delaying digital signals with no degradation and it is possible to achieve fairly sophisticated interpolation without involving undue cost and complexity.

The equipment proposed in this report is not intended as a design prototype for future digital line-store converters, but rather as a research tool to enable subjective tests to be carried out with the purpose of determining the practical limitations which can be imposed on the complexity of the

interpolation whilst achieving a high standard of output picture quality.

2. Theoretical considerations

2.1. A definition of interpolation aperture

Interpolation is necessary in standards conversion because the scanning of the original scene samples the picture vertically at a rate (in terms of spatial frequency) which depends on the number of lines per field. If a television picture is to be obtained as if generated on another standard, the original scene must first be recovered from the input samples to effectively give a continuous function which can then be re-sampled, and hence rescanned.

According to Nyquist's sampling theorem, if a signal is sampled at a rate greater than twice the highest frequency it contains, it can, in theory, be recovered perfectly from the samples using a suitable filter. However, it can, in practice, be recovered with various degrees of accuracy by replacing each sample, represented as an impulse, by an aperture function whose origin coincides with the impulse position and whose magnitude is scaled in proportion to the strength of the impulse. The value of the recovered signal at any point is then the sum of the various aperture functions corresponding to the train of samples. A representation of this process is shown in Fig. 1 where the

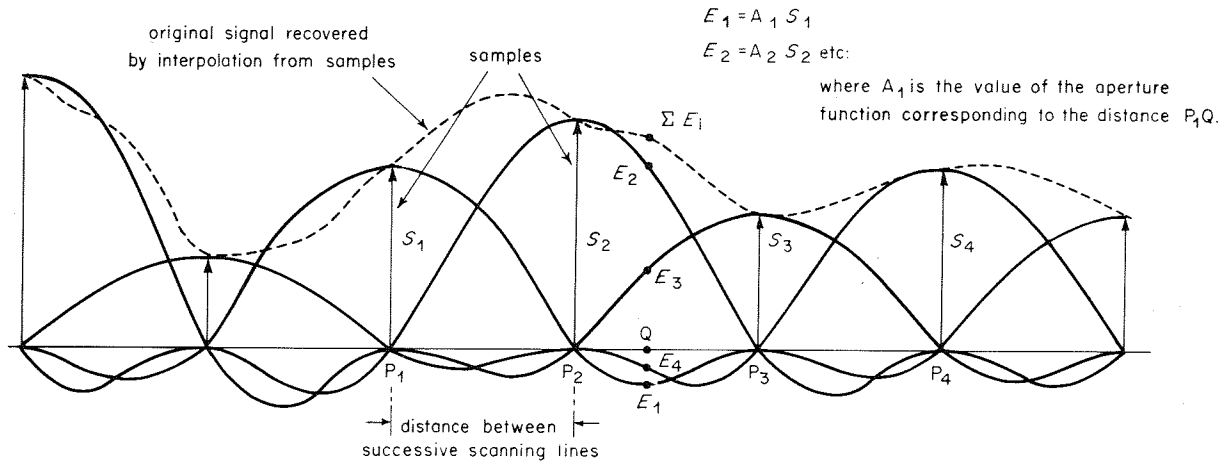


Fig. 1 - Interpolation as a summation of displaced aperture functions. The recovered signal at Q is the sum of E_1, E_2, E_3 and E_4

original signal has been recovered by interpolation from the samples. It will be observed that the aperture function used here as an example resembles $(\sin x)/x$ over a limited range. The choice of a suitable practical aperture will be discussed in Section 2.5.

The aperture function need not be symmetrical about its origin; but if it is, an alternative approach is to consider it centred on the point at which the interpolated value is required. The magnitudes of the neighbouring impulses are then scaled by the relative value of the aperture at the various impulse positions and the scaled values summed to produce the required interpolated value as shown in Fig. 2.

The process of displacing and weighting a function is a convolution operation and thus interpolation can also be described as multiplication in the frequency domain where the spectrum of the sampled signal is simply multiplied by the spectrum of the interpolation aperture. The action of sampling a signal causes its spectrum to appear as sidebands centred on multiples of the sampling frequency as in Fig. 3. The spectra centred on the sampling frequency and its harmonics can, however, be almost completely rejected if the spectrum of the interpolation aperture approximates the pass-band of a low-pass filter that would transmit only the zero-order spectrum representing the original signal.

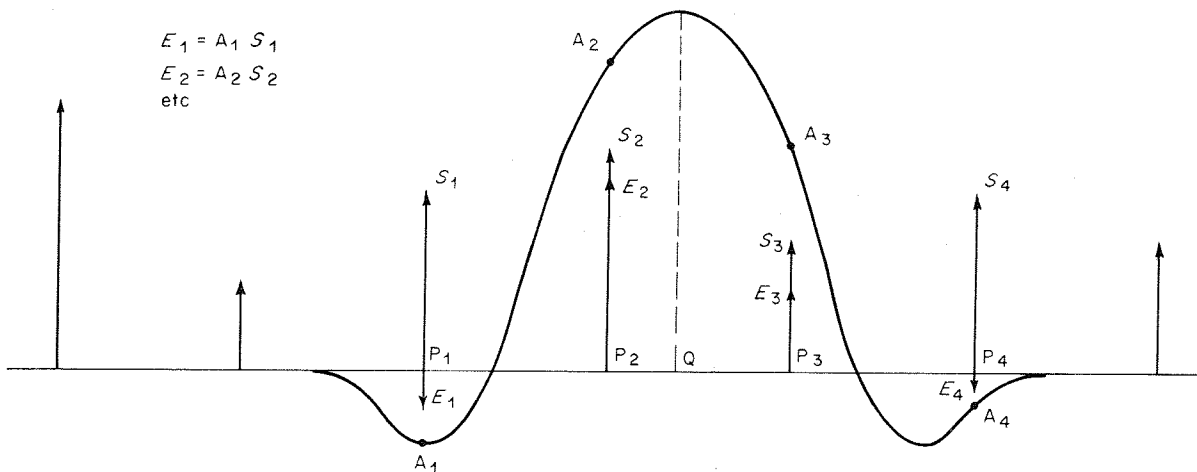


Fig. 2 - Interpolation as a summation of weighted samples. The disturbance at Q is equal to the sum of E_1, E_2, E_3 and E_4

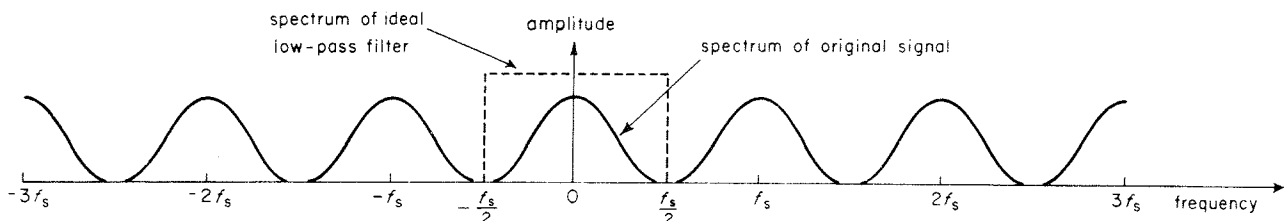


Fig. 3 - Spectrum of signal sampled at a frequency f_s

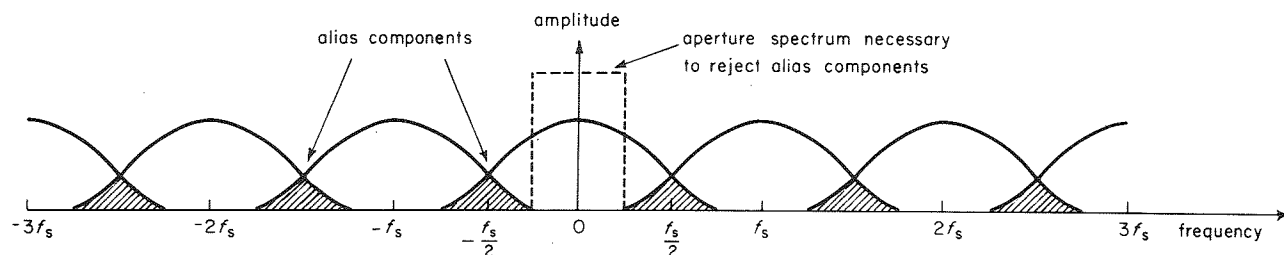


Fig. 4 - Alias components generated when original signal contains spectral components at frequencies greater than $f_s/2$

2.2. Space-time duality

The signal representing the brightness down a vertical line in the scene has spatial frequency components and is spatially sampled. Thus distance can be measured in terms of picture height (p.h.) or line height (l.h.) and frequency in cycles/p.h. or cycles/l.h. However, there is a complete analogy with the time domain in that, for a particular element position along a scanning line, the samples occur at regular intervals of time (line period) and the interpolation aperture becomes the temporal impulse response of a low-pass filter. This is the approach used in the design of analogue line-store converters² now in service.

2.3. The ideal aperture

From Fig. 3 the ideal interpolation aperture would seem to be that which has the rectangular spectrum of an ideal low-pass filter cutting at half the sampling frequency, but other factors may influence the choice.

- (1) The original scene may contain spectral components at frequencies greater than half the sampling frequency, and these components will be accentuated if excessive vertical aperture correction has been employed. The process illustrated in Fig. 3 will then produce overlapping of the repeated spectra, an effect known as 'aliasing' since the spectral components overlaid are not distinguishable from the signal components. In such cases it may be desirable to reduce the cut-off frequency of the aperture spectrum to eliminate the alias components as in Fig. 4.
- (2) The vertical bandwidth will be too high if interpolation is followed by down-conversion. In this case it may be desirable that the cut-off should be reduced below half the input sampling frequency.
- (3) The effect of interlace.

This third factor is by far the most important and is discussed further.

The following argument applies to down-conversion when both input and output standards are interlaced.

Suppose that the spatial spectrum of the brightness signal down a vertical line in the original scene is as shown in (a) of Fig. 5. If the scanning action of the source at the input standard takes place with a spatial frequency f_i c/p.h. assuming that the source aperture is infinitely small the spectrum of the sampled signal is shown in (b); as can be seen, higher order spectra are produced. The spectrum of

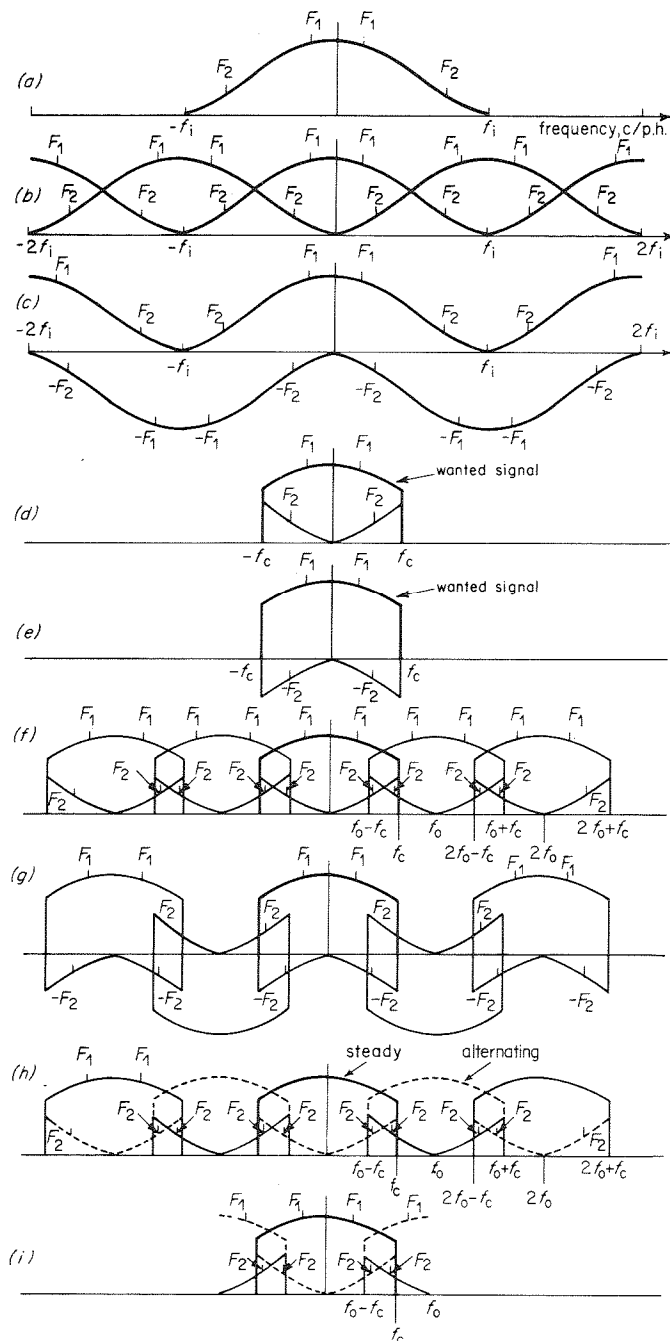


Fig. 5 - The spectral behaviour of sampled, interpolated and resampled signals

- (a) Original signal
- (b) Sampled signal
- (c) Interlace-sampled signal
- (d) and (e) Result of interpolating (b) and (c) with an 'ideal' aperture
- (f) and (g) Resampled and interlace-resampled signals
- (h) Addition of resampled and interlace-resampled signals
- (i) Resampled signal subjected to restricted bandwidth

the second, interlaced, field has inverted components (as explained in Reference 4) for odd orders as shown in (c). If spectra (b) and (c) are combined to form a complete picture at the input standard the even orders reinforce each other and the odd orders partially cancel each other out to an extent which depends on the temporal response of the display phosphor. Hence, integrated over a complete picture the effective value of f_i is doubled. However, because the eye is able to resolve temporally the individual fields, the partial cancellation of the odd orders gives rise to the familiar phenomenon of 'strob'ing' at low input frequencies (e.g. a plain raster) and 'twittering' at high frequencies (e.g. edges).

Suppose, however, that spectra (b) and (c) are interpolated and resampled in an interlaced manner before being combined. If the interpolation aperture has a rectangular spectrum cutting at f_c where $f_c < f_i/2$ the spectra of the interpolated fields are shown in (d) and (e).

If the interpolated signals are now resampled with a spatial frequency f_o , the spectra of the two fields are shown in (f) and (g). When spectra (f) and (g) are combined to form the complete output picture as in (h) steady and alternating components are produced as outlined above.

Finally, a limit must be imposed on the range of displayed frequencies because of visual acuity or the existence of line broadening³ caused by the finite display spot size. A reasonable assumption is that these effects restrict the spectral content of the displayed picture to an upper frequency of f_o as shown in (i).

The final result can be illustrated by examining the output for a sinusoidal input whose frequency varies from zero to f_i . This can be done by tracing through the products derived from a particular spectral component in Fig. 5(a). For an input frequency f in the range zero to f_c indicated by F_1 in Fig. 5(a) an alternating component at $f_o - f$ is produced in addition to the steady component at f . If f lies between f_c and $f_i - f_c$ no output is possible but if f lies beyond $f_i - f_c$ as indicated by F_2 in Fig. 5(a) a steady and an alternating spurious component are again produced. As the dotted lower curves in Fig. 5(i) are parts of the curve of Fig. 5(a) translated by an amount f_i and the solid lower curves are parts of the same curve translated by an amount $(f_i - f_o)$ the alternating component is at a frequency $f_i - f$ and the steady component is at a frequency $f - (f_i - f_o)$. This behaviour is summarised in Fig. 6. The alternating component at $f_o - f$ in region (1) would be present in a signal generated at the output standard and is therefore inherent. If, in region 3, the steady spurious component is the more objectionable and its visibility decreases with increasing frequency then there might be an advantage in lowering f_c , thereby narrowing region 3 and delaying the onset of the steady component. This would be paid for by a loss in definition. On balance there does not seem to be any strong reason for not putting f_c equal to $f_i/2$. It must be emphasised, however, that although it is possibly the best solution it is not ideal because of the spurious products in region 3 and therefore the result of the conversion process field by field can never be the same between interlaced systems as that obtained by generation at the output

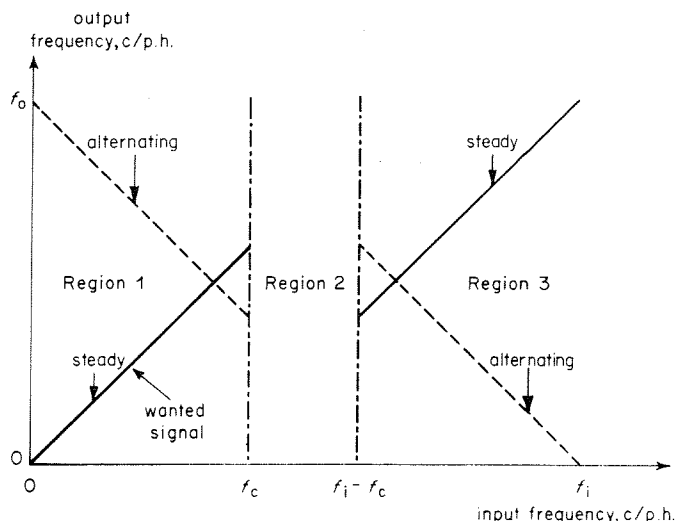


Fig. 6 - The spectral behaviour of the converted display using an aperture having a rectangular spectrum cutting at f_c where $f_c < f_i/2$

standard at the source. The aperture profile having this flat spectrum cutting off at $f_i/2$ is of $(\sin x)/x$ form, having zeros spaced at the distance between successive lines of an input field.

2.4. Limitations on the practical aperture

If the best aperture is of $(\sin x)/x$ form it is clear that a practical aperture of limited width and having an imperfect spectrum approaches the 'ideal' as its width increases. There are, however, certain conditions of varying importance which should preferably be satisfied by any aperture of limited width. These conditions are discussed below.

- (i) The aperture spectrum must have zeros at integer multiples of the input sampling frequency f_i . If this is so the higher orders of a sampled constant signal (corresponding to a picture having uniform vertical brightness) are completely rejected and no spurious components are produced. Failure to meet this condition would, after resampling, give rise to horizontal striations on a uniform background. This condition is also equivalent to the requirement that values of the aperture profile spaced apart a distance of one field line should sum to a constant regardless of their position, an extension of Equation 2 in Reference 4.
- (ii) The value of the constant in condition (i) should be unity if the interpolation process has unity gain.
- (iii) The spectrum of the aperture should have zeros at integral multiples of the output sampling frequency so that no spurious product is formed at zero frequency except when the input frequency is f_i . In general, a spurious component at zero frequency can only occur by the translation in the resampling process after interpolation of a frequency at an integral multiple of f_o . Thus if the aperture spectrum has zeros at these frequencies, no spurious dc component can result.

- (iv) The value of the normalised spectrum at half the input sampling frequency should approximate to 0.5, corresponding to the ideal case. This point is discussed further in Appendix A.
- (v) The aperture profile should have no discontinuity at the ends. See, for example, Fig. 2. Failure to meet this condition with apertures having a width not equal to an integral number of lines implies a discontinuity in output signal as the position of the output line varies, assuming a uniform d.c. input signal. With apertures having a width equal to an integral number of lines no discontinuity of output signal occurs, but an extra sample is needed to produce the output when an output line coincides with an input line.
- (vi) The gradient of the profile should be zero at the ends (except for any discontinuity) if it is continuous elsewhere (Fig. 2). Failure to meet this condition with apertures of both integral and non-integral line widths implies a discontinuity of gradient at points on the profile spaced at integral line widths from the ends.

2.5. The derivation of a practical aperture

There are several possible approaches to the problem of deciding on the profile of a practical aperture. The approach described here is a limited Fourier series method, an extension of a method already described elsewhere.⁴

The starting point is a theorem⁵ which states that an aperture of width w units has a spectrum the value of which can only be independently specified at discrete frequencies which are multiples of $1/w$ cycles/unit. The spectrum in between these frequencies is derived by the superposition of appropriately scaled $(\sin x)/x$ functions centred on each frequency, and having zeros $1/w$ cycles/unit apart. This follows because the aperture can be expressed as the product of a repetitive waveform having a period of w units and a rectangular pulse of width w units and the spectrum of the aperture is the convolution of the spectra of these two functions. This process is illustrated in Fig. 7 for an aperture width w equal to $6/f_i$ p.h. where $1/f_i$ is the input sampling spatial period. The spectra of the repetitive waveform and the pulse are shown in (a) and (b) respectively and the spectrum of the aperture in (d). As can be seen the spectrum parameters A_0, A_1 etc. are the amplitudes of the Fourier components of the repetitive waveform, at the frequencies 0, $1/w$ etc. Thus the aperture profile can be expressed as the Fourier series

$$S(\theta) = A_0 + A_1 \cos \theta + A_2 \cos 2\theta \dots, |\theta| < \pi \quad (1)$$

In Equation (1) θ is an angular coordinate which varies from $-\pi$ to $+\pi$ across the aperture and is given by

$$\theta = 2\pi d/w \quad (2)$$

where w is the aperture width in p.h. and d is the distance coordinate in p.h. measured relative to the centre of the aperture.

Fig. 7 is drawn for an aperture width equal to an integral number of lines but the method of building up the spectrum applies equally well to non-integral line widths. When the width is an integral number of lines, so that

$$w = (n + 1)/f_i \quad (3)$$

where n is the order of interpolation,

then the spectrum has the value $A_{n+1}/2$ at the frequency f_i which can thus be specified as zero thereby satisfying condition (i) of Section 2.4.

Moreover, to simplify the mathematics, the values of A_r for $r > n+1$ can also be set to zero thereby ensuring that the spectrum passes through zero at the points $rf_i/(n+1)$ for $|r| \geq n+1$. Thus the aperture profile becomes a limited Fourier series given by

$$S(\theta) = A_0 + A_1 \cos \theta + A_2 \cos 2\theta \dots + A_n \cos n\theta$$

$$= \sum_{r=0}^n A_r \cos r\theta, |\theta| < \pi \quad (4)$$

and the aperture spectrum $F(\nu)$ is given by

$$\begin{aligned} F(\nu) &= A_0 \operatorname{sinc} \nu w + \frac{A_1}{2} \left[\operatorname{sinc} (\nu - 1/w)w + \operatorname{sinc} (\nu + 1/w)w \right] + \dots \\ &\quad + \frac{A_n}{2} \left[\operatorname{sinc} (\nu - n/w)w + \operatorname{sinc} (\nu + n/w)w \right] \\ &= \sum_{r=0}^n \frac{A_r}{2} \left[\operatorname{sinc} (\nu w - r) + \operatorname{sinc} (\nu w + r) \right] \end{aligned} \quad (5)$$

where $\operatorname{sinc} \nu w = \sin(\pi \nu w)/(\pi \nu w)$ is the spectrum of a rectangular pulse of width w .

2.6. Optimisation of the aperture

The profile defined by Equation (4) satisfies condition (i) of Section 2.4 because A_{n+1} is zero and condition (vi) because it is a finite series. However it does not necessarily satisfy the remaining four conditions as can be seen by referring to Fig. 8 which shows an aperture of width 4 lines and the problem of optimisation is to decide on the importance of the various conditions consistent with obtaining a spectrum as close as possible to the 'ideal' rectangular form of cutting at $f_i/2$. If there are insufficient degrees of freedom because n is too small it will be impossible to impose all the conditions simultaneously.

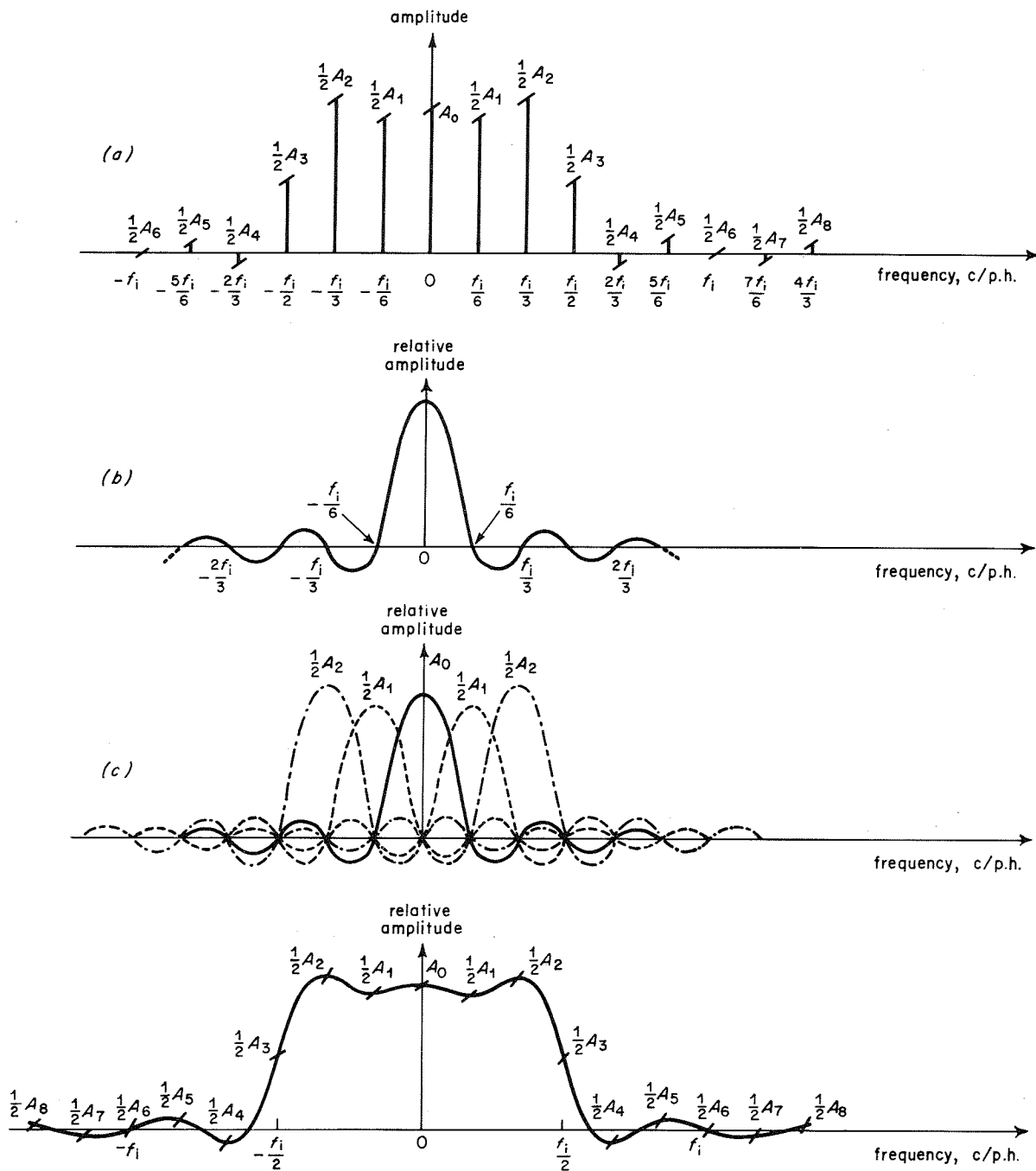


Fig. 7 - Derivation of aperture spectrum of width $6/f_i$ p.h.

(a) Spectrum of repetitive waveform (b) Spectrum of single square pulse of width $6/f_i$
 (c) Some components of aperture spectrum (d) Aperture spectrum obtained by summation of all components

Assuming $n > 2$ so that all the conditions can be accommodated there are four linear equations relating $A_0, \dots, A_n$ and the spectrum is optimised as a function of the remaining independent parameters.

The optimisation can be performed by minimising an error function which represents the deviation of the actual spectrum from the ideal spectrum. An error function which has been used in theoretical work was formed by summing values of the spectrum expressed in decibels at regular frequency intervals within the passband (i.e. up to $f_i/2$) and adding to this a similar contribution from the stopband but expressed as a deviation from a desired value of -40 dB.

Alternatively the parameters $A_0, \dots, A_n$ can be given arbitrary values and the spectrum and profile inspected to see how closely the four conditions are obeyed. If necessary the parameters can then be adjusted. This second approach was found to be more successful particularly as it was found in practical work that some of the conditions could be relaxed.

2.7. Spatial quantisation of the aperture

The ratio of 625 to 405 line frequencies is such that 81 output lines occur in the same time interval as 125 input

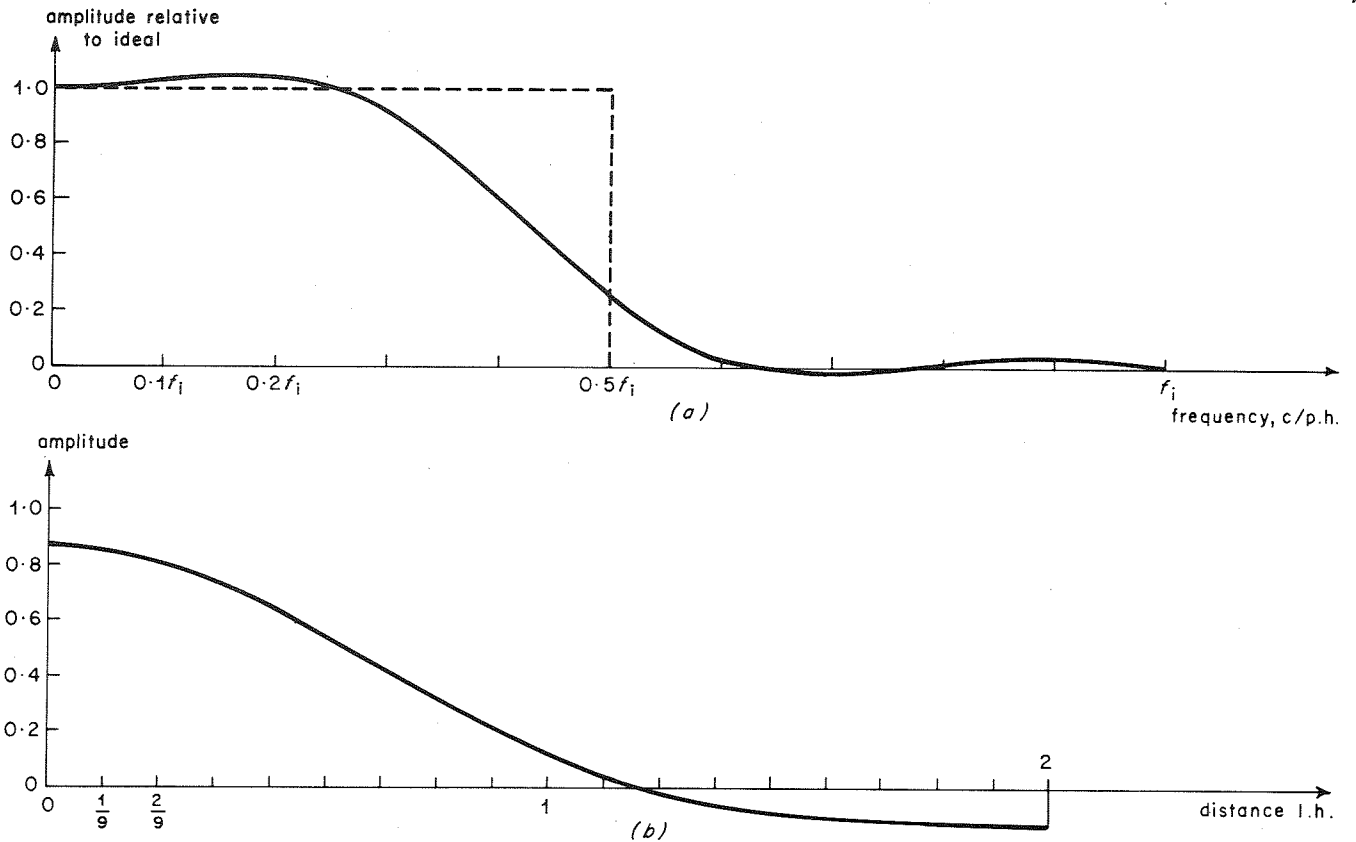


Fig. 8 - Typical aperture spectrum and profile obtained by optimisation process
(a) Spectrum (b) Profile

lines. This relationship can also be assumed to hold for the spatial positions of the lines. This means that only 81 discrete values of the interpolation aperture profile per line of each input field need be known. If, however, the aperture is several lines wide the number of values that needs to be stored may become prohibitively large and, moreover, if the values are to be set independently as part of a subjective experimental arrangement the complexity of the apparatus reaches impractical proportions.

The required number of stored values can be reduced if certain restrictions are imposed on the interpolation aperture. Firstly the aperture can be assumed symmetrical so that the number of required values is halved. Secondly, the apertures can be spatially quantised into less than 81 discrete values although impairments in interpolation will then result in addition to those caused by the finite width of the aperture. The form of these impairments can easily be analysed in the frequency domain.

The quantising can be expressed as a sampling action followed by a boxcar operation (Fig. 9). Thus, if the aperture is quantised into k quanta per input line height it is equivalent to sampling with a frequency $f_q = kf_i$ c/p.h. and forming each sample into a rectangular pulse of width $1/kf_i$ p.h. In spectral terms if the original aperture spectrum is $F(\nu)$ the quantisation produces a spectrum

$$F_q(\nu) = \text{sinc } \nu/kf_i \sum_{t=-\infty}^{\infty} F(\nu + tkf_i) \quad (6)$$

where $\text{sinc } u = \sin(\pi u)/(\pi u)$.

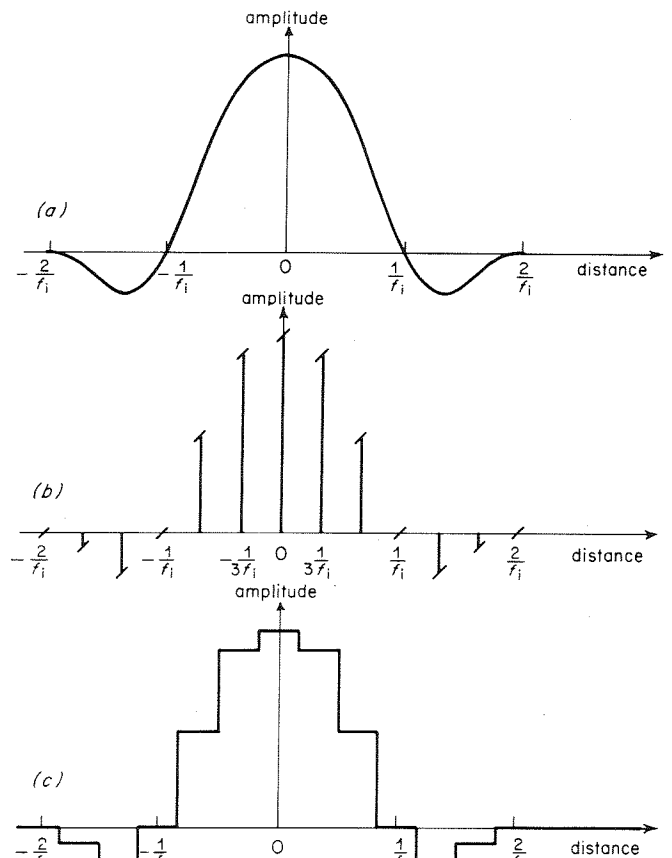


Fig. 9 - Derivation of spatially quantised aperture for $k = 3$
(a) Original aperture (b) Sampled aperture (c) Quantised aperture

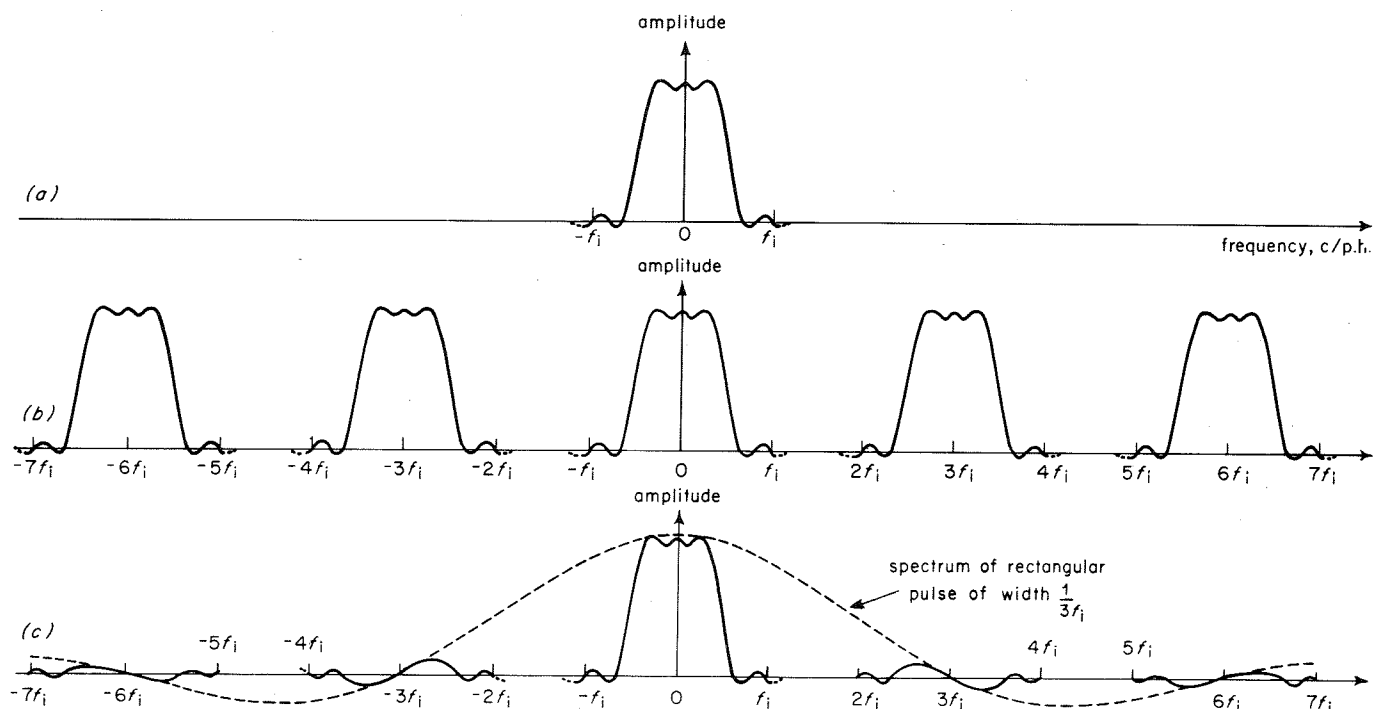


Fig. 10 - Derivation of the spectrum of a spatially quantised aperture for $k = 3$
 (a) Original spectrum (b) Spectrum of sampled aperture (c) Spectrum of quantised aperture

The summation term results from the sampling, and the sinc term from the boxcar operation. The derivation of a typical form of $F_q(\nu)$ is shown in Fig. 10. As can be seen the effect of quantisation is to introduce additional spectral lobes at the frequencies $\pm kf_i, \pm 2kf_i$, etc.

Now the first scanning process (at the input frequency f_i) produces components centred on 0, $\pm f_i, \pm 2f_i$ etc (Fig. 5) and thus significant energy exists near the frequencies $\pm kf_i, 2kf_i, \dots$ after interpolation. The effective resampling process described in Section 2.3 (at the output frequency f_o) translates all components through integral multiples of f_o and thus the energy in the additional lobes may eventually produce substantial components near the origin. It is shown in Appendix B that the result of the two sampling processes, with or without intervening interpolation, produces additional spurious components at spatial frequencies separated from the input frequency by integral multiples of $f_i/125 (= f_o/81)$. If the component at $f_r (= f + rf_i/125)$ is termed the r th resultant order the amplitude of this component, $e(f_r)$, is given by

$$e(f_r) = \sum_{s=-\infty}^{\infty} F_q(f + [35r + 81s] f_i) \quad (7)$$

This equation also holds for unquantised apertures when F_q becomes F but because of the absence of the additional lobes, the level of the spurious products is much lower. It is also shown in Appendix B that Equation (7) reduces to the unquantised case when $k = 81$. This must be so if interpolation requires values of the aperture only at the centre of each quantum. This argument can be extended to cover the case when k is any multiple of 81.

The energy pattern of resultant orders repeats every 81 so that the 81 orders nearest the origin contain all the information about the levels of the spurious products. The effect of quantisation can now be assessed by comparing the level of these 81 products with and without quantisation.

The physical significance of the resultant orders is best appreciated by considering an input signal in the form of sloping bars of sinusoidal cross-section for example the corner bars of a Test Card. It is shown in Appendix B that the spurious components also appear as sloping bars as shown in Fig. 11.

The interaction of the spurious bars with the input bars gives rise to the familiar phenomenon known as 'knotting'.

For example, when linear interpolation is used the level of the relevant resultant order causing the strongest spurious component is about -20 dB and this is visible on the corner bars of a Test Card as can be seen in Fig. 12.

2.8. Theoretical results obtained

Aperture spectra have been optimised according to the criterion mentioned in Section 2.6 for various integral values of n up to 7, that is for aperture profiles up to 8 lines wide. In general, increasing the value of n gives a sharper cut, as might be expected. For example, with $n = 7$ the spectrum falls from 90% to 10% of the value at the origin in a frequency range of $0.11f_i$ compared with $0.32f_i$ for the present analogue converters.*

* This information was communicated to the authors by Mr. R.E. Davies.

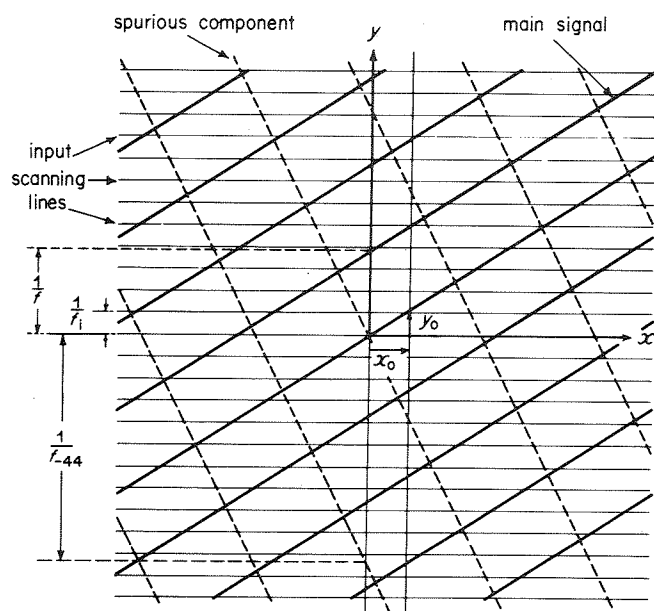


Fig. 11 - Positions of maximum brightness on the display for an input of sloping bars showing the -44th order spurious component. The output scanning lines are not shown

To obtain theoretically satisfactory rejection of unwanted signals it was found necessary to relax condition (iv) of Section 2.4, allowing the spectrum to take on values much lower than 0.5 at the frequency $f_i/2$. In the absence of condition (iv) however, the optimisation resulted in spectra having a very good stopband response but a poor passband response.

The second approach of setting arbitrary parameter values was then tried. The imposition of condition (v) was found to be most restricting and when this was removed spectra were produced which were theoretically and experimentally acceptable and which generally satisfied condition (iii) quite closely. Finally condition (ii) was relaxed very slightly to allow for the fact that the aperture values were

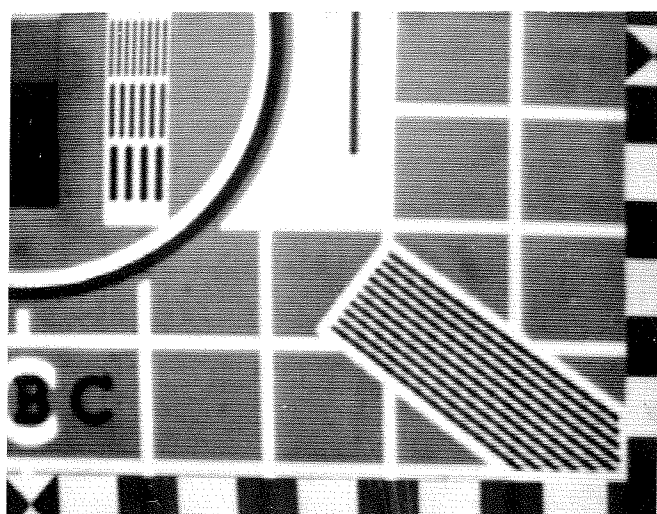


Fig. 12 - Photograph illustrating interpolation errors in the corner bars of Test Card C

quantised in amplitude and therefore did not always sum to unity. Fig. 8 shows a typical aperture spectrum and profile obtained.

As a subsidiary investigation certain profiles were quantised into 9 and 3 quanta per line and the resulting spurious products calculated and compared with those from the unquantised aperture and the quantised ideal aperture. A summary of the results is shown in Appendix C where components down to the -40 dB level are included.

It is seen in Appendix C that the behaviour of an aperture having a width of 8 lines is virtually identical to that of an infinite-width aperture. Moreover, it can be shown that for 9 quanta per line the spurious products for an infinite-width aperture are at least 29 dB down. As products at -20 dB cause an impairment of about grade 3 (see Section 2.7) it is considered that an aperture profile having 9 quanta per line should be acceptable.

2.9. The effect of contrast law on interpolation

The above theoretical treatment is based upon one important assumption which is not necessarily upheld, namely that the chosen interpolation aperture will operate upon signals having a linear relationship to brightness. In practice, standards converters are located at transmitter stations, and so receive an input composite signal which has been gamma-corrected.

Standards conversion of gamma-corrected signals will generally result in an incorrect summation of aperture samples, these errors being greatest when the difference in brightness from line to line is greatest.

A series of exploratory tests was carried out using the digital converter with a linear interpolation aperture and suitable switching arrangements were provided whereby conversion could take place before or after gamma correction, and the pictures produced by the two processing paths were viewed alternately. Slides covering a good cross section of subjects were selected (including those known to show up interpolation errors), and a suitable high quality slide scanner was used as a source of signals. However, very little or no improvement could be detected when converting signals before gamma correction.

It was concluded that for the majority of subjects, the line to line differences in picture content were restricted by the vertical resolution of the system, and were consequently never large enough to show up the inaccuracies caused by operating on gamma-corrected signals. It was provisionally concluded, therefore, that gamma correction was unlikely to influence the design of the interpolation system, although further confirmatory tests will be carried out when more sophisticated interpolation methods are available.

3. Practical proposals

To assess the practical importance of the theoretical conclusions reached above, subjective tests must be per-

formed on the visible impairments to the television picture caused both by simple and also by more refined interpolation methods. For this work a flexible and comprehensive interpolator is being developed which will provide immediate comparison of the same programme material interpolated in various ways.

Fig. 13 shows the basic block diagram of the proposed interpolator. The tapped delay will consist of seven one-line digital delays so that signals from eight adjacent field lines of the picture may be simultaneously available for processing. The eight video signals will be multiplied by appropriate coefficients in eight multipliers and the resulting products added, with due regard to sign, in the adder.

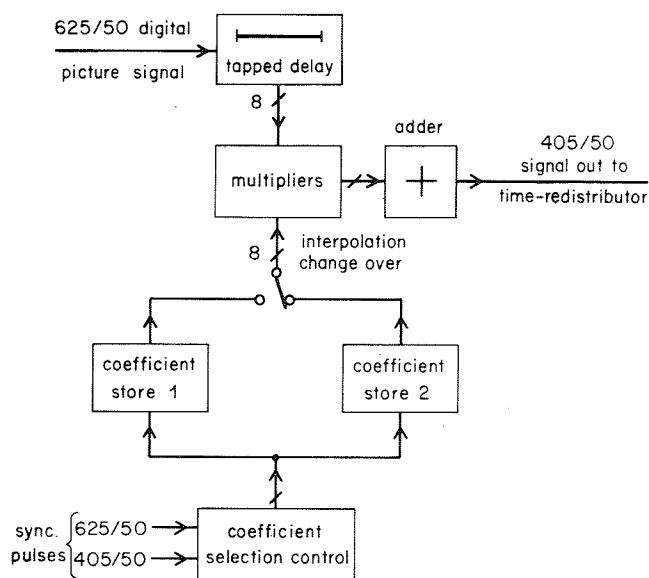


Fig. 13 - Basic block diagram of proposed interpolator

The complexity of the equipment increases rapidly as the width of the aperture, the number of bits in the coefficients, and the fineness of the spatial quantising of the aperture are increased. For example, an aperture eight lines wide, with eight bits per coefficient and nine quantised positions, requires an interpolator using nearly 1500 simple integrated circuits with correspondingly high cost. For an experimental interpolator this degree of complexity has therefore been taken as the upper limit.

Immediate comparison between two chosen sets of interpolation parameters will be provided by the interpolation changeover switch which will select coefficients from one or other of two coefficient stores in use. A number of plug-in store boards will be available by means of which comparisons can be quickly made.

By suitable choice of the coefficients loaded into a store, the width, shape, amplitude resolution and to some extent the spatial quantisation of the aperture may be varied. It is hoped that a set of interpolation parameters will be found which are both subjectively satisfactory and simple to instrument in operational converters.

4. Conclusions

The best aperture spectrum for standards conversion between interlaced television systems is probably rectangular, cutting at half the input standard field spatial frequency. The corresponding aperture profile is of $(\sin x)/x$ form having zeros one field line apart. A practical aperture can approach this ideal as closely as desired as its width increases. If the aperture is expressed as one cycle of a limited Fourier series the resulting spectrum is a function of a limited number of variables and can be optimised subject to certain constraints. These constraints are that the spatial spectrum should have zeros at integral multiples of the input and output spatial scanning frequencies and fall to half amplitude at half the input spatial scanning frequency. In addition the aperture profile and its gradient should both fall to zero at the ends of the aperture. If the aperture is spatially quantised to save storage of information the smallest number of quanta per input-field line that gives acceptable degradation probably decreases with increasing aperture width. For a width of 8 input lines 9 quanta is probably acceptable. The minimum width of aperture, amplitude resolution and number of spatial quanta per line which are acceptable remain to be determined subjectively. To this end an experimental interpolator is being constructed which, it is hoped, will provide a means for exploring the subjective effect of varying all aperture parameters. The results of these tests, giving recommendations to Designs Department, will be the subject of a further report.

5. References

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APPENDIX A

The Transmission of a Sharp-Cut Filter at the Cut-Off Frequency and Related Theorems

Suppose a d.c. signal is sampled so that its spectrum consists of lines at integral multiples of the sampling frequency, f_s . If this signal is passed through a filter having a rectangular spectral response cutting at $\pm f_c$ where $f_c < f_s$ then only the zero-order spectral component is transmitted and the output is d.c. In the time domain the output of the filter can be expressed by convolution as the superposition of $(\sin x)/x$ pulses of unit height having $T_c/2$ between successive zeros and spaced apart a distance T_s . If, therefore, $T_s = aT_c/2 = aT_z$ the output of the filter is given by

$$e_o(t) = \sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + naT_z)/T_z}{\pi(t + naT_z)/T_z} = \text{constant for } 0 < a < 2 \quad (8)$$

In particular for $a = 1$ each peak of a pulse coincides with the zeros of all the others and therefore e_o at the peaks must be unity. Thus

$$\sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + nT_z)/T_z}{\pi(t + nT_z)/T_z} = 1 \quad (9)$$

From energy considerations the value of the constant in Equation (8) must be proportional to f_s and therefore inversely proportional to a . Thus

$$\sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + naT_z)/T_z}{\pi(t + naT_z)/T_z} = \frac{1}{a} \text{ for } 0 < a < 2 \quad (10)$$

When $a = 2$ pulses occur at intervals of $2T_z$ and consequently each peak again coincides with zeros. Thus e_o is unity at the points $0, 2T_z, 4T_z, \dots$. However, e_o is zero at the points $T_z, 3T_z, \dots$ because pulse zeros occur at these points. Further, e_o must be symmetrical about the points $0, 2T_z, 4T_z, \dots$ and $T_z, 3T_z, \dots$ because of the symmetry of the pulses and therefore the gradient must be zero at all these points. Turning to the spectral domain, when $a = 2, f_s = f_c$ and consequently the first-order spectral lines fall at the cut-off frequency. Together with the previous argument it is clear that e_o involves a d.c. and a sinusoidal term such that

$$e_o(t) = \frac{1}{2}(1 + \cos 2\pi f_c t) \quad (11)$$

The corresponding Fourier transform of e_o is

$$\frac{1}{2} \{ \delta(\nu) + \frac{1}{2} [\delta(\nu - f_c) + \delta(\nu + f_c)] \} \quad (12)$$

where $\delta(\nu)$ is the Dirac-Delta function, showing that the spectral lines at $\pm f_c$ have half the amplitude of that at the origin. Thus, if the spectrum of the filter is given by

$$F(\nu) = 1, |\nu| < f_c \quad (13)$$

$$F(\nu) = 0, |\nu| > f_c$$

the value of $F(\nu)$ at $|\nu| = f_c$ is 0.5. Any real filter which approximates to $F(\nu)$ should, therefore, also approximate to the value of 0.5 at the cut-off frequency.

When $2 < a < 4$ the first-order spectral lines fall inside the pass-band and thus the output of the filter is

$$e_o(t) = \text{constant} (1 + 2 \cos 2\pi t/T_s) \quad (14)$$

For $a = 3, T_s = 3T_z$ and consequently each peak coincides with zeros. Thus $e_o = 1$ at the points $0, 3T_z, 6T_z, \dots$

Therefore, for $a = 3$

$$\sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + n3T_z)/T_z}{\pi(t + n3T_z)/T_z} = \frac{1}{3} (1 + 2 \cos 2\pi t/3T_z) \quad (15)$$

Again from energy considerations the value of the constant in Equation (14) must be inversely proportional to a , thus

$$\sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + naT_z)/T_z}{\pi(t + naT_z)/T_z} = \frac{1}{a} (1 + 2 \cos 2\pi t/aT_z) \quad \text{for } 2 < a < 4 \quad (16)$$

When $a = 4$ the second-order spectral lines fall at the cut-off frequency and therefore a further term must be included in the right-hand side. In general, by a similar argument it can be shown that for $a \neq 2p, p$ integral

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + naT_z)/T_z}{\pi(t + naT_z)/T_z} &= \frac{1}{a} (1 + 2 \cos 2\pi t/aT_z + \\ &+ 2 \cos 2\pi 2t/aT_z + \dots + 2 \cos 2\pi mt/aT_z) \\ &= \frac{1}{a} (1 + 2 \sum_{n=1}^m \cos 2\pi nt/aT_z) \end{aligned} \quad (17)$$

where m is the integer next below $a/2$.

For $a = 2p, p$ integral

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \frac{\sin \pi(t + naT_z)/T_z}{\pi(t + naT_z)/T_z} &= \\ &= \frac{1}{a} (1 + 2 \sum_{n=1}^{p-1} \cos 2\pi nt/aT_z + \cos 2\pi t/2T_z) \end{aligned} \quad (18)$$

Equations (17) and (18) are useful in that they enable an infinite series to be expressed in terms of a finite series.

APPENDIX B

Spurious Products Resulting from Two Successive Sampling Operations

B1. Derivation of the products

Suppose a sinusoidal signal of frequency f is sampled at a frequency f_i . The resulting spectrum consists of pairs of spectral lines centred on the points nf_i (at $nf_i \pm f$) where n is the first sampling order. If this sampled signal is then resampled at a frequency f_o each component of the spectrum is repeated with a period of f_o so that the resulting spectrum consists of pairs of lines centred on the points $nf_i + mf_o$ where m is the second sampling order.

$$\text{Suppose also that } f_o/f_i = p/q \quad (19)$$

where p and q are integers without common factor. Also let

$$f_o = ph \quad f_i = qh \quad (20)$$

so that $h = f_i/q$ is a conveniently small unit in terms of which to measure frequency differences. Then the resulting (output) spectrum consists of pairs of lines centred on the points $(nq + mp)h = rf_i/q$, say where r is an integer.

Now it is well known that if p, q have no common factor, then integers m_o, n_o always exist so that

$$m_o p + n_o q = 1 \quad (21)$$

and it is also clear that these integers are not unique, since m_o can be replaced by $m_o + \lambda q$ and n_o can be replaced by $n_o - \lambda p$, where λ is any integer. Further, for any value of r we can find integers m_1, n_1 such that

$$m_1 p + n_1 q = r \quad (22)$$

In fact, we can take

$$m_1 = rm_o - sq \quad n_1 = rn_o + sp \quad (23)$$

where s is any integer, and the n_1 th first sampling order will contribute to the r th resultant order if n_1 has any value satisfying (23).

If interpolation follows the first sampling process, the amplitude of the n th first sampling order is simply

$$F(nf_i + f) + F(nf_i - f)$$

where $F(\nu)$ is the interpolation aperture spectrum at spatial frequency ν . Hence the r th resultant order consists of lines at frequencies $rf_i/q \pm f$ or $rh \pm f$, and the total output is

$$e(rh \pm f) = \sum_{s=-\infty}^{\infty} F(n_1 f_i \pm f) \quad (24)$$

where n_1 is given by (23).

If the aperture is quantised into k quanta per field line, $F(\nu)$ is given by Equation (6) instead of by Equation (5) and the expression for the r th resultant order becomes

$$e(rh \pm f) = \sum_{s=-\infty}^{\infty} \left\{ \text{sinc}[(n_1 f_i \pm f)/(kf_i)] \times \sum_{t=-\infty}^{\infty} F[(n_1 + kt)f_i \pm f] \right\} \quad (25)$$

where, as before,

$$h = f_i/q \quad n_1 = n_o r + ps.$$

For 625/405 conversion, we are mainly concerned with the particular case

$$p = 81; \quad q = 125; \quad m_o = -54; \quad n_o = 35 \quad (26)$$

B2. Symmetry considerations

As the modulus of any spectrum is symmetrical about the origin and all spectra considered are real

$$e(rh - f) = e(-rh + f) \quad (27)$$

The left-hand term is the r th order product resulting from the spectral line at $-f$ and the right-hand term is the $-r$ th order product resulting from the line at $+f$. Thus all the spectral line amplitudes can be obtained by considering the products only from the line at $+f$. Further if

$$f_r = rh + f < 0$$

there will be a product of equal amplitude at $-f_r$. Thus considering only positive frequencies, products occur at the frequencies $|f_r|$. Thus Equation (25) becomes

$$e(|f_r|) = \sum_{s=-\infty}^{\infty} \left\{ \text{sinc}[(n_1 f_i + f)/kf_i] \times \sum_{t=-\infty}^{\infty} F[(n_1 + kt)f_i + f] \right\} \quad (28)$$

B3. Special cases

For $k = 81$

$$\sum_{t=-\infty}^{\infty} F[(n_1 + kt)f_i + f] = \sum_{t=-\infty}^{\infty} F[(35r + 81t)f_i + f]$$

which is independent of s . Thus

$$e = \left\{ \sum_{s=-\infty}^{\infty} \text{sinc} [(n_1 f_i + f)/81 f_i] \right\} \times \left\{ \sum_{t=-\infty}^{\infty} F[(35r + 81t)f_i + f] \right\} \quad (29)$$

But the first factor of e in Equation (29) is of the form

$$\sum_{n=-\infty}^{\infty} \text{sinc} (t + nT)/T \text{ which}$$

by Equation (9) is equal to unity. Thus

$$e = \sum_{t=-\infty}^{\infty} F[(35r + 81t)f_i + f] = \sum_{s=-\infty}^{\infty} F(n_1 f_i + f)$$

which is the unquantised case above.

Similarly it may be shown that the case $k = 162$ reduces to the unquantised case.

The 'Ideal' Aperture

$$F_o(v) = 1 \quad 0 < |v| < f_i/2$$

$$F_o(v) = 0 \quad |v| > f_i/2$$

For $|f| < f_i/2$,

$$\sum_{t=-\infty}^{\infty} F_o[(n_1 + kt)f_i + f] = 1 \text{ for } n_1 + kt = 0 \\ = 0 \text{ otherwise.}$$

The condition for F_o to be different from zero may be written

$$t = -(n_1/k) = -(35r/k) - (81s/k) \quad (30)$$

As t is an integer there are two special cases.

- (1) $k = 3, 9$, or 27 , s has any value, r is a multiple of k . Then only the $0, k, 2k, \dots, ik$ orders are non-zero and are given by

$$e = \sum_{s=-\infty}^{\infty} \text{sinc} [(35ik + 81s)f_i + f]/kf_i$$

which by Equation (17) can be written

$$e = \frac{k}{81} \left[1 + 2 \sum_{s=1}^t \cos 2\pi s(35ikf_i + f)/81f_i \right] \quad (31)$$

where t is the integer next below $81/(2k)$.

- (2) $k = 5, 7$ or 35 , r has any value, s is a multiple of k . Then all orders are non-zero and if $s = 0, k, 2k, \dots, ik, \dots$ the r th order is given by

$$e = \sum_{i=-\infty}^{\infty} \text{sinc} [(35r + 81ik)f_i + f]/kf_i$$

which by Equation (17) can be written

$$e = \frac{1}{81} \left[1 + 2 \sum_{s=1}^{40} \cos 2\pi s(35rf_i + f)/81kf_i \right] \quad (32)$$

B4. The effect of interlace

When the first and second sampling processes are both interlaced the odd orders produced by each process are inverted for the interlaced field. Thus for the interlaced field an odd second-sampling order generated from an even first-sampling order is inverted but an odd second-sampling order generated from an odd first-sampling order is not inverted. The various alternatives are summarised in Table I.

Now considering the relationship

$$81m + 125n = r$$

the parity is summarised in the truth table, Table II. Comparing Table I and Table II it can be seen that the even resultant orders are steady whereas the odd resultant orders alternate during a complete picture.

TABLE I

m	n	Resultant
O	O	S
O	E	A
E	O	A
E	E	S

TABLE II

m	n	r
O	O	E
O	E	O
E	O	O
E	E	E

O Odd
E Even
A Alternating
S Steady

B5. General observations

The general expression for e is best solved by computer. The infinite summations are limited by placing a limit on the modulus of the argument of sinc and F . In practice limits of 10 and $10f_i$ were found satisfactory.

In general for unquantised apertures the strength of the first sampling orders decreases monotonically as n increases. Thus the strongest resultant orders are those

which contain contributions from first sampling orders near the origin, that is, low values of n . For example for $n = 1$ the first sampling order is at $f + f_i$ and contributes to the resultant order at this frequency, namely the 125th resultant order. By similar reasoning, it is possible to predict the sequence of resultant orders in descending strength. Because the energy pattern of resultant orders repeats every 81 it is sufficient to quote the order number of lowest modulus. The sequence is

$$0, \pm 37, \pm 7, \pm 30, \pm 14, \pm 23, \pm 21 \dots$$

This sequence breaks down for quantised apertures because the strength of the first sampling orders does not decrease monotonically due to the presence of the additional spectral lobes.

B6. The physical significance of the products

The analysis performed so far has involved real spectra corresponding to horizontal bars of sinusoidal cross-section symmetrically disposed about the horizontal axis. Under these conditions the presence of the spurious components produces an amplitude modulation of the original signal. However, the effect of the spurious components is more easily appreciated by considering sloping bars of sinusoidal cross-section as in Fig. 11.

Without loss of generality the origin can be placed at the intersection of a bar peak and an input scanning line making the vertical spectrum of the sampled signal down the line $x = 0$ real as before. For an arbitrary value of x say x_o the vertical signal suffers a linear shift y_o where

$$y_o = a_o x_o \quad (33)$$

and a_o is the slope of the bars.

If the vertical frequency of the bars is f the linear shift imparts a phase shift of $-2\pi f y_o$ to the components $n f_i + f$ and $+2\pi f y_o$ to the components at $n f_i - f$ in the sampled signal. If the signal is now resampled, with or without interpolation, and a scanning line passes through the origin then each component is repeated with a period f_o as before. Thus the theory already developed holds except that all products resulting from the line at $\pm f$ suffer a phase shift of $-2\pi f y_o$ and vice versa. Moreover, the symmetry conditions apply so that considering only positive frequencies, products occur at $|f + n f_i + m f_o| = |f + r f_i / 125| = |f_r|$ with a phase such that if $f_r > 0$ the phase is negative and vice versa. For $f_r > 0$ the phase shift of $-2\pi f y_o$ produces a linear shift of $f y_o / f_r$ and therefore the slope of the spurious product is $f y_o / f_r x_o = a_o f / f_r$. For $f_r < 0$ the phase shift of $+2\pi f y_o$ produces a linear shift of $-f y_o / |f_r| = f y_o / f_r$ as before. Thus for all values of f_r the spurious component has a vertical frequency of $|f_r|$ and a slope a given by

$$a = a_o f / f_r = a_o / (1 + r f_i / 125 f) \quad (34)$$

For example the corner bars of Test Card C have a vertical frequency of 80 c/p.h. and a slope of 0.75. As $f_i = 312\frac{1}{2}$ c/p.h. Equation (34) becomes

$$a = \frac{0.75}{1 + r/32} \quad (35)$$

From Section B5 the strongest spurious component occurs for $r = 37, -44, -125$ etc. The more objectionable effect occurs for even values of r which produce steady spurious components. Thus taking $r = -44$, $a = -2$ which gives a pattern having a slope somewhat greater than the orthogonal value of $-4/3$ and having a coarse vertical frequency of 30 c/p.h. This pattern is visible when linear interpolation is used although the level of the -44 th resultant order is about -20 dB (calculated from the spectrum of the linear interpolation aperture).

APPENDIX C

Strength of Resultant Orders for an Input Frequency of $0.3 f_i^{(1)}$

Width	Quanta per line	81	9	3
	dB relative to input	resultant order number	resultant order number	resultant order number
Infinite	0 to -10 -10 to -20 -20 to -30 -30 to -40	0	0 -9, +9 +18, -18, +27 -27, +36, -36	0 +30, -30, -21, +21, +9 -9, +39, -39, -12, +12, -3, +3, +27, -27 -24, +24, +6, -6, -36, +36, +15, -15, +33, -33
8 lines ⁽²⁾	0 to -10 -10 to -20 -20 to -30 -30 to -40	0	0 -9, +9 +18, -18, +27 -27, +36, -36 ⁽³⁾	0 +30 -30, -21, +21, +9, -9 +39, -39, -12, +12, -3, +3, +27, -27 -24, +24, +6, -6, -36, +36, +15, -15, +33, -33 ⁽³⁾
2 lines with 405 zero	0 to -10 -10 to -20 -20 to -30 -30 to -40	0 -37, -7 +37, +7, +30, -30, -14 +14, +23, -23, -21, +21	0 +37 -37, -9, -7, +9, +7 +18, -18, +30, +27, -27	0 +30, -30, +37, -21 +21, +9, -9, +39, -39 -12, +12, +7, +18, -18
2 lines without 405 zero	0 to -10 -10 to -20 -20 to -30 -30 to -40	0 +37	0 +37 -9, +9, +18, -18, +28 -35, +27, -27, +36, -36	0 +37 +30, -30, +7, -21, +21 -14, +9, -9, -23, +39, -39, +16, -12 +12, +28, +18, -18, -35, -33, +33, -2

NOTES

1. This frequency gives most spurious components for the 3 quanta case
2. Each order strength is 0.5 dB greater than the infinite width case
3. +37th order also significant at a frequency of $0.4 f_i$